THE PATCH CONSTRUCTION IS THE SAME THING AS ALGEBRAIC DCPO REPRESENTATION

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TALK OUTLINE.

A. DESCUBE ALGEBRAIC DEPO REPRESENTATION TOPOLOGICALLY.

B. RECALL PATCH CONSTRUCTION.

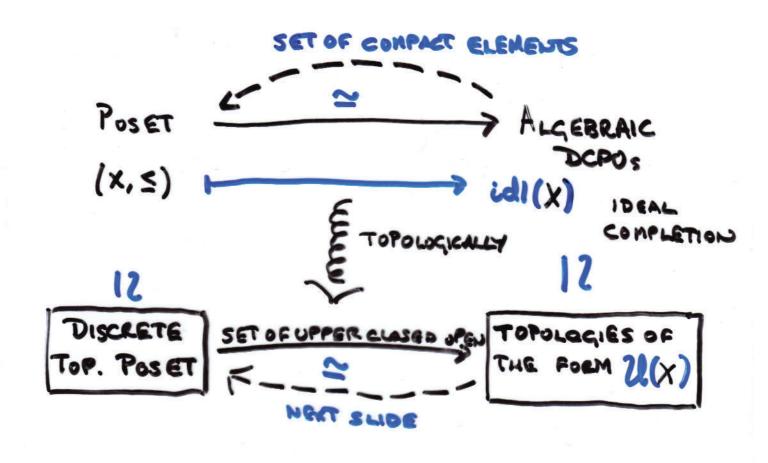
C. TOPOLOGICAL PRODUCT XXY WIA SUPLATTICE TENSOR WIA PREFRAME TENSOR

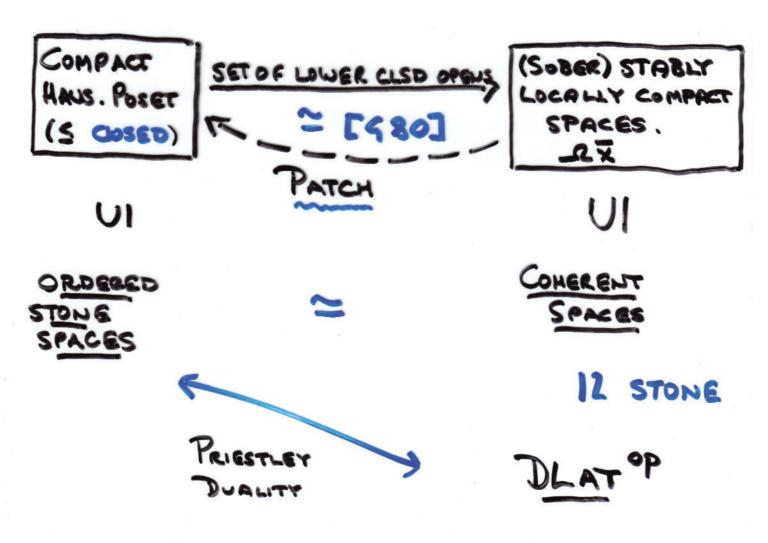
D. RELATIONAL COMPOSITION AS SUPLATTICE HOMOMORPHISM PREFRAME HOMOMORPHISM

E. ALGEBRAIC DORO REPRESENTATION VIA SUPLATTICE Homs.

PAIN FUL BUT ...

- F. DUAL ARGUMENT VIA PREFRAME HONS. GIVES PATCH
- G. CATEGORICAL ACCOUNT.
- H. FURTHER WORK: AXIOMATIC ACCOUNT OF STABLY LOCALLY COMPACT.





PX U(X°TXX) U(X°TXX) E WHEER I IS IDEMPOTENT. NEW FOR TODAY. (1) CAN CONSTRUCT U(X°TXX) AND I (AND HENCE PX) ONLY BY REFERENCE TO U(X). (2) EXACTLY THE SAME CONSTRUCTION WORKS ON LIX THE OPENS OF A STABLY LOCALLY COMPACT X, GIVING PATCH.

I WE CAN IN FACT RECOVER PX AS AN EQUALIZER

GIVEN A POSET (X, S) CAN WE RECOVER P(X) FROM U(X)?

PX SPLITS SINCE

$$\Delta^{1}(J \times \hat{I}) = \{j(i,j) \in U \ \text{lix} \hat{I}\} = I$$

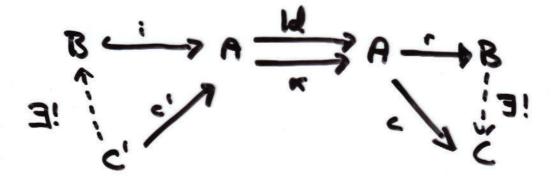
$$\sum_{i \in I} \sum_{j \in I} \sum_{i \in$$

Example. (X, S) A POSET. J

$$U(X^{q}xX) \longrightarrow P(XxX) \xrightarrow{\Delta'} PX \xrightarrow{\exists_{A}} P(XxX) \xrightarrow{\lfloor X \hat{l}} P(XXX)$$

 $R \longrightarrow S; (R \cap A); \leq$

IDEMPOTENT.



THIS MAKES & BOTH AN EQUALIZER (INCLUSION) AND COEQUALIZER (QUOTIENT): THAT IS YC,C'

IF K: A-JA HAS KEEK THEN IT SPLITS IF. Bi: B-JA AND r: A-JB WITH ir = K ri= Id.



GIVEN SPACES X, Y, DESCRIBE XXY.

ETHER SET UQV = {(U,V) LEU, VEV}

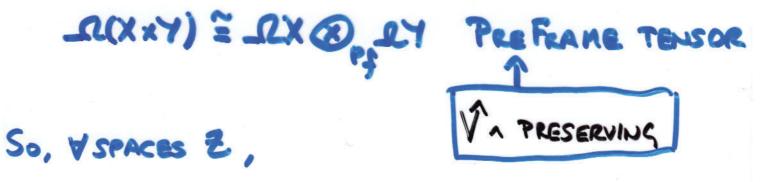
ALL UELRX VELY AS GENERATORS THEN

LR(XXY) = IX OLY SUP-LATTICE O.

THEN FOR ALL SPACES 2, V PRESERVING. SUP (IX & IX) = Sup (IX, Sup (IX, IZ))

 $\frac{QR}{(=(U^{C}QV^{C})^{C})} = \frac{Q}{(U^{C}QV^{C})^{C}} = \frac{Q}{(U^{C}Q$

THEN ON THESE GENERATORS ...



PREFR (IX @ IY, IZ) = PREFR (IX, Pref (IY, IZ)

TECHNICAL LEMMAS.

Lemma I: FOR DISCRETE X,Y IL(XXY) = SUP(IX, IY) AND RELATIONAL COMPOSITION MAPS TO FUNCTION COMPOSITION . Lemma 2: FOR COMPACE HAUSDORFF X, Y _R(XXY) = Prefr (RX, RY) AND REL. COMPOSITION MAPS TO EUNCTION COMP. PROOF 1 REXXY (arra; R) $(1\otimes \psi)(\Delta) \leftarrow \psi$ PROOF Z $R' \leq XXY \longmapsto (a \mapsto (a'; R)))$ $(10+)(\Delta^{c}) \leftarrow + \Psi$

APPLICATION CREATING THE OPENS OF ALGEBLAIC DCP03. (X, S) POSET P(XXX) => SUP(PX,PX) $\le \longrightarrow \uparrow$ UX IS THE SPLIT OF THE IDEMPOTENT 1. / APPLICATION GIVEN POSETS (X, S) AND (Y, S) U(X ** X) = SUP(UX,UY) PX => PX -> UX>UY -> PY => PY So SUP(UX, XY)= F +: PX → UY + + + F \cong $\{\psi: \mathcal{P}X \rightarrow \mathcal{P}Y \mid \psi \uparrow = \psi, \uparrow \psi = \psi\}$ = { R = X × Y | S; R= R= R; S} EU(X*xX) COROLLARY Sup(UX, R) = U(X OP) (Y=1) So ev: UX * @U(X) ->_R ev(IOI)=1 (=) FLEIDI. // CORBLLARY U(X *) @U(X) = U(X * XX)

FINALLY : NOW TO RECOVER PX FROM UX.
RECALL
$PX \longrightarrow \mathcal{U}(X^*X) \longrightarrow \mathcal{U}(X^*X)$
2 ≤;(-04);≤ 2
$u(x^{*})\otimes u(x) \xrightarrow{iJ} Sup(ux, ux)$
TRANSPOSE:
$\Xi: \mathcal{U}(x^{\varphi}) \otimes \mathcal{U}(x) \otimes \mathcal{U}(x) \longrightarrow \mathcal{U}(x) \cong SUP(\mathcal{U}x^{\varphi}, \Omega)$
AGAIN
$\overline{\overline{T}}$: $u(x^{q}) \otimes u(x) \otimes u(x) \otimes u(x^{q}) \longrightarrow \mathcal{I}$
How ACONT = (I @I@J@J)=ev(InJ@IN)
VES R SILOADIS SXXXXXX HAS
$\mathcal{X}_{R}(i,j,\overline{i},\overline{j})=1 \iff (\overline{i},\overline{j}) \in \leq j(j x j) \land \Delta) \leq \leq j(j x j) \land \Delta = 1 $
=> =k Tsksj jsksi.
ev(lintj@ijoit)=1 => 3 kelintjoijoit
$ \stackrel{\text{So}}{=} P_X \longrightarrow \text{Sup}(\mathcal{U}_X, \Omega) \otimes \mathcal{U}_X \xrightarrow{\mathcal{U}} \text{Sup}(\mathcal{U}_X, \mathcal{V}_X) \\ \xrightarrow{\mathbb{T}} \mathbb{T} $
WHERE I IS THE TEANSPOSE OF AN
EVALUATION MAP.

THE SAME THING FOR COMPACT HAUS. (X, 5) IL(X X) = Prefo (RX, ex) ≦ ┣━> 介叩

IZ is (DEFINITION) SPLITTING OF APP. For (X, S), (Y, S) _ R(X ** XY) = Prefr(RX, RY) So * PREFR (IT, I)=IX. (1=1) * ev: IX => @ IX -> I * IX * Ør LX = I(X * XX) AGAIN THE ACTION (X XX) - > D(X XX) REI-> (S; ROA; S)"

IS TRANSPOSE OF D. (TY) Og SIZ Og SIZ Og SIX) -> -2 IO IO JOJHJev (IVJOIVJ)

JUST AS IN DISCRETE CASE LEFT CHECKING ...

A(X**XX) ~ AXQ AX -7 AX -> A(XXX) -> Q(XXX)

SPLITS TO AX.

I.E. A" "ON" VA(U) =U ANY OPEN UELOX.

TAKE COMPLEMENTS THIS IS: (S; (JAC); S) OA=C ALL CLOSED C. ... or Escardo

ALG-DCPU -> POSC.

WHOSE ORDER DUAL IS ALC. DOPO REP.

STLOCK C -> KHAUSPOS C

THERE IS A FUNCTOR PATCH

(=) XEC " IS DISCRETE.

XEC COMPACT HAUSDORFF

THIS AKIOMATIZATION,

YES. WE CAN AXIOMATIZE A FRAGMENT OF TOPOLOGY AS AN ORDER ENRICHED CATEGORY C. THEN, WITH RESPECT TO

DISCRETE. PREFRAME = SUPLATTICE.

OBJECTIONS: COMPACE HAUSDORFF

Do I REALLY MEAN THE SAME?

FLAVOUR OF THIS AXIOMATIZATION .

* I HAS FINITE PRODUCTS

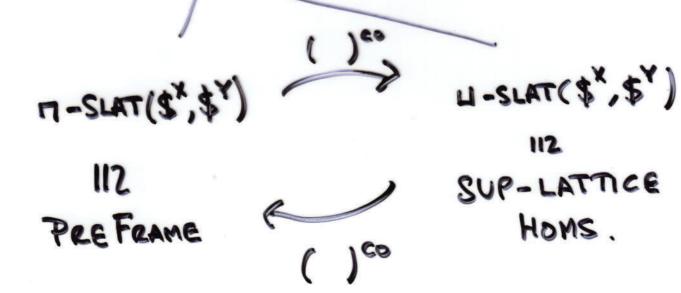
* C ORDER ENRICHED

BISTRIBUTIKE LATTICE.

 $(\Delta - | \Pi, u - | \Delta)$

THEN C = TOP

Tor (\$*,\$*) = depo (IX, IY)





RECALL X COMPACT <=> $\Omega!: \Omega \longrightarrow \OmegaX$ HAS A PREFRAME RIGHT ADJ.

AXIONATICALLY XEC COMPACT (=) \$:\$ -> \$ HAS A RIGHT ADJOINT.

Similarly (VERNEULEN) X is compace Hausdorff (Compace &) _RA: R(XXX) - ZXX HAS PREFRAME RIGHT ADJOINT AND $V_A(a V RA(I)) = V_A a V I.$

DUALLY X is DISCRETE (=) AD HAS LEFT ADJOINT JA: RX -> RKXX) WITH

IABE = ((I) DR AB) E

(JOYAL & TIERNEY).

=) ORDER DUAL CONCEPTS.

AND DUALLY (ii) ALGEBRAIC DCPO.

THIS ENSURES THE RUNCTOR (X,S)HOX IS ESSENTIAL SURJECTIVE BY DEFINITION NICER TO HAVE AN INTRINSIC DEFINITION OF (i) STABLY LOCALLY COMPACT

OF) ALGEBRAIC DEPOS.

THIS MIRRORS THE DEFINITION OF THEOPOUS

OUR "DEFINITION" HAS BEEN (=) IIX IS A SPLITTING OF IX I II

(a=VJb|becag) AND IEEL AND acebi, b= =) acebi, b=

(=) IIX CONTINUOUS POSET

IN FACT A X STABLY LOCALLY COMPACT

WANT AN INTRINSIC DEFINITION OF STABLY LOCALLY COMPACT.

FURTHER WORK.

SUMMARY

- * ALGEBRAIC DOPO REPRESENTATION CAN BE VIGWED AS AN ACTION ON TOPOLOGIES.
- WIEWED AS SUCH IT IS THE SAME CONSTRUCTION AS PATCH.
- HOMS AND PREFRAME HOMS.
- * THE US TO REPRESENT RELATIONS ON DISCRETE SPACES AS SUPLATTICE HOMS & CLOSED RELATIONS ON COMP. HAUS. SPACES AS PREFRAME HOMS.
- BY STATING THE RESULTS RELATIVE TO A SUITABLY A KIDMATIZED ORDER ENRICHED CATEGORY (C.